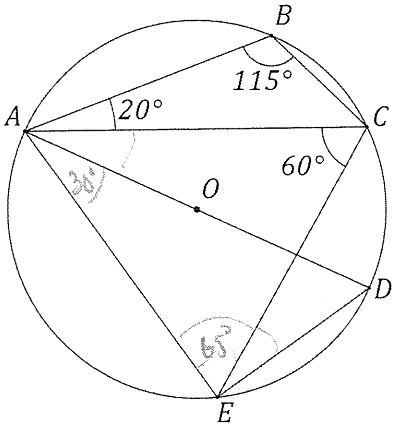
Section Two: Calculator-assumed **65% (98 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 8 **(5 marks)**

Points B, C and E lie on the circle with diameter AOD as shown below. $\angle ABC = 115°$, $\angle BAC = 20°$ and $\angle ACE = 60°$.



Determine the size of the following angles.

(a) $\angle ADE$. (1 mark)

60°

(b) $\angle EAD$. (1 mark)

30°

(c) $\angle AEC$. (1 mark)

65°

(d) $\angle CAD$. (1 mark)

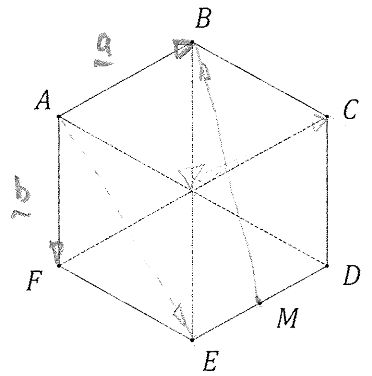
25°

(e) $\angle CED$. (1 mark)

25°

Question 9 **(7 marks)**

(a) $ABCDEF$ is a regular hexagon. The midpoint of side DE is M.



Let $\mathbf{a} = \overrightarrow{AB}$ and $\mathbf{b} = \overrightarrow{AF}$. Express each of the following in terms of $\mathbf{a}$ and $\mathbf{b}$.

(i) $\overrightarrow{BC}$. (1 mark)

$\underline{a}+\underline{b}$

(ii) $\overrightarrow{AE}$. (1 mark)

$\underline{b}+\underline{a}+\underline{b} = \underline{a}+2\underline{b}$

(iii) $\overrightarrow{MB}$. (1 mark)

$-\frac{\underline{a}}{2}-2\underline{b}$

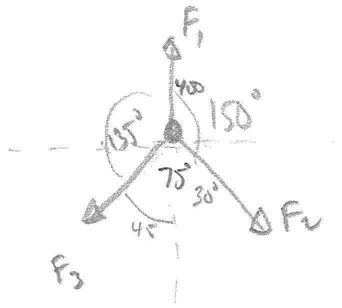
$= -\frac{1}{2}\underline{a}-2\underline{b}$

(b) Three forces, F_1, F_2 and F_3 act on a body that remains in equilibrium.

F_1 has a magnitude of 400 N. The angle between the directions of F_1 and F_2 is 150°, between F_1 and F_3 is 135° and between F_2 and F_3 is 75°.

Determine the magnitudes of F_2 and F_3,

(4 marks)



$\updownarrow$ $400 - (F_3 \cos 45° + F_2 \cos 30°) = 0$

$\leftrightarrow$ $F_2 \sin 30° = F_3 \sin 45°$

Solving $F_2 = 293N$, $F_3 = 207N$

Question 10 8 (7 marks)

(a) A number is to be formed by randomly selecting three **different** digits from those in the number 93265. Determine how many different numbers

(i) start with an odd digit. (1 mark)

$\boxed{3 \mid 4 \mid 3} = 36$

(ii) end with an even digit. (1 mark)

$\boxed{4 \mid 3 \mid 2} = 24$

(iii) start with an odd digit or end in an even digit. (2 marks)

$36 + 24 - \boxed{3 \mid 3 \mid 2}$

$= \underline{42}$

(b) A computer user has forgotten their six character, case-sensitive password, but know that they always use a permutation of F, F, 1, 9, 9, and 9 - their initials and the year they were born. Determine how many passwords are possible if

(i) the F's must both be uppercase. (2 marks)

$$\frac{6!}{\underset{'f'}{2!} \times \underset{'9'}{3!}} = 60$$

(ii) either F can be lowercase or uppercase. (~~1~~ mark) 2

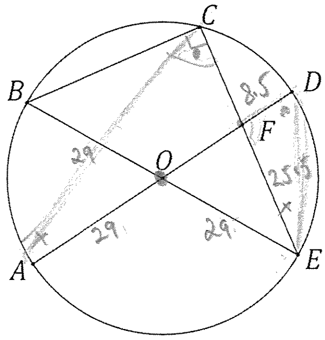
FF, Ff, fF, ff ie 4

$\therefore 4 \times 60 = \underline{240}$

Question 11 **(8 marks)**

$\sqrt{29}$
14.5

(a) Triangle BCE is such that B, C and E lie on a circle with centre O and radius 29 cm. Diameter AD and chord CE intersect at F, so that $DF = 8.5$ cm and $EF = 25.5$ cm. Determine the lengths OF, CF and BC. (5 marks)



14.5
8.5
6.0

$\Delta DFE \sim \Delta CFA$ (AA)
$\angle DFE = \angle CFA$ (vert opp)
$\angle CAD = \angle DEC$ (angles on same arc)

$OF = 29 - 8.5$
$= 20.5$ cm

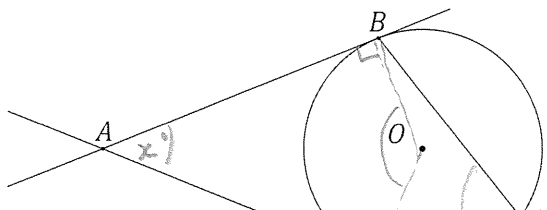
$\frac{8.5}{CF} = \frac{25.5}{49.5}$

$CF = 16.5$ cm

25.5
16.5
42.0

$BC = \sqrt{58^2 - 42^2}$
$= 40$ cm

(b) In the diagram below, points B, C and D lie on a circle with centre O. The tangents to the circle at B and D intersect at point A. If $\angle BAD = x$, prove that $\angle BCD = 90° - \frac{x}{2}$. (3 marks)



$\angle BOD = 360° - (180 + x)$
$= 180 - x$ $\therefore \angle BCD = \frac{1}{2}(180 - x)$
$= 90 - \frac{x}{2}$ (angle at centre = 2x angle at circumference)

Question 12 (~~9~~ 10 marks)

Transformation A is an anti-clockwise rotation about the origin of 90° and matrix $B = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$.

(a) Represent transformation A as a 2×2 matrix. (2 marks)

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

(b) Describe the transformation represented by matrix B. (2 marks)

dilation factor 2 in x-direction
" " 3 " y "

(c) Determine the coordinates of the point $P(-15, -11)$ following transformation A and then transformation B. (~~2~~ 3 marks)

$$BA = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 3 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -2 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} -15 \\ -11 \end{bmatrix} = \begin{bmatrix} 22 \\ -45 \end{bmatrix}$$

$\therefore$ coords are $(22, -45)$

(d) Following transformation B and then transformation A, point Q is transformed to point $Q'(12, 7)$.

Determine the single matrix that will transform Q' back to Q and hence determine the coordinates of point Q. (3 marks)

$$AB = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 0 & -3 \\ 2 & 0 \end{bmatrix}$$

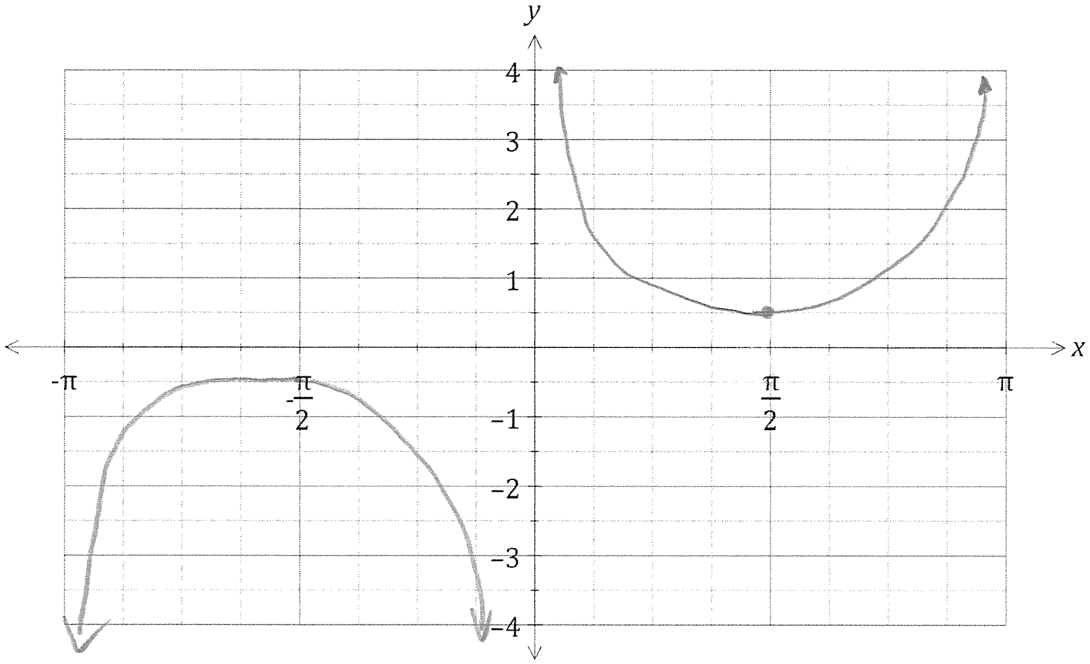
$$\begin{bmatrix} 0 & -3 \\ 2 & 0 \end{bmatrix}^{-1} = \frac{1}{6}\begin{bmatrix} 0 & 3 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2} \\ -\frac{1}{3} & 0 \end{bmatrix}$$

$$Q = \begin{bmatrix} 0 & \frac{1}{2} \\ -\frac{1}{3} & 0 \end{bmatrix}\begin{bmatrix} 12 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} 7/2 \\ -4 \end{bmatrix}$$ $\therefore$ coords of $Q\left(\frac{7}{2}, -4\right)$

Question 13 **(8 marks)**

(a) On the axes below sketch the graph of $y = \frac{1}{2}\sec\left(x - \frac{\pi}{2}\right)$. $\frac{1}{2\cos(x-\frac{\pi}{2})}$ (3 marks)



(b) Consider the function $f(t) = 2\sin t - 5\cos t$, $t \geq 0$.

(i) $f(t)$ can be expressed in the form $r\sin(t - \alpha)$, where $r > 0$ and $0 \leq \alpha \leq \frac{\pi}{2}$. Determine the values of r and α, rounded to 2 decimal places. (3 marks)

$r\sin(t-\alpha) = r\sin t\cos\alpha - r\cos t\sin\alpha \Rightarrow r\cos\alpha = 2$, $r\sin\alpha = 5 \Rightarrow r = \underline{5.39}$, $\alpha = \underline{1.19}$

(ii) Hence or otherwise determine the minimum value of $f(t)$ and the smallest value of t for this minimum to occur. (2 marks)

$f(t) = 5.39\sin(t - 1.19)$ $\therefore$ min value $= -5.39$ $(-\sqrt{29})$

$t = 5.90$ approx

Question 14 9 ~~(8 marks)~~

(a) Consider the vectors $\mathbf{p} = (24, -143)$ and $\mathbf{q} = (20, -21)$. Determine

(i) the angle between the directions of vectors $\mathbf{p}$ and $\mathbf{q}$. (1 mark)

angle = 34.1°

(ii) two vectors that are perpendicular to $\mathbf{q}$ and have the same magnitude as $\mathbf{p}$. 4 ~~(3 marks)~~

$|\underset{\sim}{p}| = 145.$

$|q| = 29$

$\hat{q} = \frac{1}{29}\begin{pmatrix}20\\-21\end{pmatrix}$

$\therefore \perp \hat{q} = \pm\frac{1}{29}\begin{pmatrix}21\\20\end{pmatrix}$

$\begin{pmatrix}20\\-21\end{pmatrix}\cdot\begin{pmatrix}a\\b\end{pmatrix} = 0$

$20a - 21b = 0$

$a = 21$, $b = 20$ or $a = -21$, $b = -20$

req'd vectors with magnitude 145 are

$\pm\frac{145}{29}\begin{pmatrix}21\\20\end{pmatrix} = \begin{pmatrix}105\\100\end{pmatrix}$

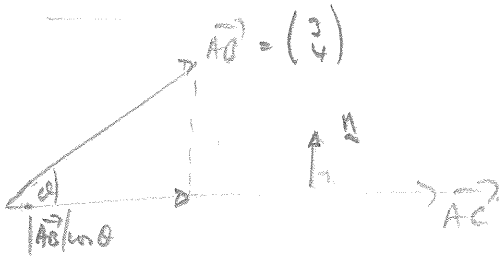
(b) If $\overrightarrow{AB} = \begin{pmatrix}3\\4\end{pmatrix}$ ~~(3, 4)~~ and $\overrightarrow{AC} = \begin{pmatrix}-2\\1\end{pmatrix}$ ~~(−2, 1)~~, determine

(i) the component of $\overrightarrow{AB}$ parallel to $\overrightarrow{AC}$. vector proj of $\vec{AB}$ on $\vec{AC}$ (2 marks)

$\hat{\vec{AC}} = \frac{1}{\sqrt{5}}\begin{pmatrix}-2\\1\end{pmatrix}$

$= \left[\begin{pmatrix}3\\4\end{pmatrix}\cdot\begin{pmatrix}-2\\1\end{pmatrix}\right]\frac{1}{\sqrt{5}}\frac{1}{\sqrt{5}}\begin{pmatrix}-2\\1\end{pmatrix}$

$= -\frac{2}{5}\begin{pmatrix}-2\\1\end{pmatrix} = \begin{pmatrix}\frac{4}{5}\\-\frac{2}{5}\end{pmatrix} = \begin{pmatrix}0.8\\-0.4\end{pmatrix}$



(ii) the component of $\overrightarrow{AB}$ perpendicular to $\overrightarrow{AC}$. ✓ (2 marks)

now $\begin{pmatrix}0.8\\-0.4\end{pmatrix} + \underset{\sim}{n} = \begin{pmatrix}3\\4\end{pmatrix}$

$\underset{\sim}{n} = \begin{pmatrix}2.2\\4.4\end{pmatrix}$

Question 15 9 ~~**(8 marks)**~~

(a) Express the recurring decimal $1.1\overline{58}$ as a rational number. ~~(2 marks)~~ 3

$x = 1.15858\ldots$

$100x = 115.858\ldots$

$100x - x = 115.8585\ldots - 1.1585\ldots = 114.7 \Rightarrow 99x = 114.7$

$x = \frac{114.7}{99} = \frac{1147}{990}$

(b) Use a counterexample to explain why the statement $(\forall x \in \mathbb{Z})(\exists y \in \mathbb{Z})(2xy = 24)$ is false. (2 marks)

integer

$x = 48 \Rightarrow 2 \times 48 \times y = 24$,

$y = \frac{24}{2 \times 48} = \frac{1}{4}$ not an integer

(c) Prove, by contradiction, that $\sqrt{6}$ is irrational. (4 marks)

assume $\sqrt{6}$ is rational

ie $\sqrt{6} = \frac{a}{b}$ where a & b are both integers with no common factors

$\Rightarrow 6 = \frac{a^2}{b^2}$

$= 6b^2 = a^2 \Rightarrow a^2 = 2(3b^2) \Rightarrow a^2$ is even

$\therefore$ a is even

if $a = 2p$ (p integer) then $(2p)^2 = 6b^2$

ie $4p^2 = 6b^2$

$\Rightarrow 3b^2 = 2p^2 \therefore 3b^2$ is even

ie b is even

this contradicts original assumption $\therefore \sqrt{6}$ is irrational

Question 16 **(7 marks)**

(a) Let the angle $\theta = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$.

(i) Use your calculator to write down an exact value for $\sin\left(\frac{\pi}{12}\right)$. (1 mark)

$$\sin\frac{\pi}{12} = \frac{\sqrt{6}-\sqrt{2}}{4} \quad \left[\frac{\sqrt{2}(\sqrt{3}-1)}{4}\right]$$

(ii) Use an angle sum or difference identity to show how to obtain the above exact value for $\sin\left(\frac{\pi}{12}\right)$. (3 marks)

$$\sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right) = \sin\frac{\pi}{3}\cos\frac{\pi}{4} - \cos\frac{\pi}{3}\sin\frac{\pi}{4}$$

$$= \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}} - \frac{1}{2}\cdot\frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{3}-1}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{2}(\sqrt{3}-1)}{4}$$

(b) Prove the identity $\sin x + \sin 2x + \sin 3x = (1 + 2\cos x)\sin 2x$. (3 marks)

$$\text{RHS} = \sin 2x + 2\cos x \sin 2x = \sin 2x + 2\sin 2x \cos x$$

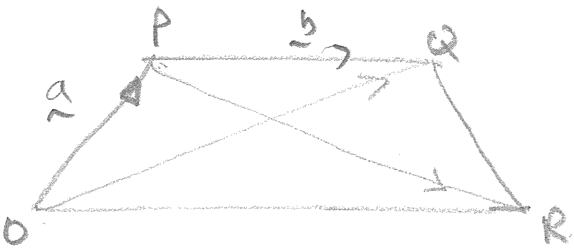
$$= \sin 2x + \sin 3x + \sin x$$

$$= \underline{\text{LHS}}$$

Question 17 **(~~9~~ 10 marks)**

Trapezium $OPQR$ has parallel sides PQ and OR such that $|\overrightarrow{OR}| = k|\overrightarrow{PQ}|$. Let $\overrightarrow{OP} = \mathbf{a}$ and $\overrightarrow{PQ} = \mathbf{b}$.

(a) Sketch the trapezium. (1 mark)



(b) Determine vectors for $\overrightarrow{OQ}$ and $\overrightarrow{PR}$ in terms of k, $\mathbf{a}$ and $\mathbf{b}$. (2 marks)

$\overrightarrow{OQ} = \underset{\sim}{a} + \underset{\sim}{b}$, $\overrightarrow{PR} = \overrightarrow{OR} - \overrightarrow{OP}$

$= k\underset{\sim}{b} - \underset{\sim}{a}$

(c) Show that the scalar product of $\overrightarrow{OQ}$ and $\overrightarrow{PR}$ is $k|\mathbf{b}|^2 - |\mathbf{a}|^2 + (k-1)\mathbf{a}\cdot\mathbf{b}$. (~~2~~ 3 marks)

$\overrightarrow{OQ} \cdot \overrightarrow{PR} = (\underset{\sim}{a} + \underset{\sim}{b}) \cdot (k\underset{\sim}{b} - \underset{\sim}{a})$

$= k\underset{\sim}{a}\cdot\underset{\sim}{b} - |\underset{\sim}{a}|^2 + k|\underset{\sim}{b}|^2 - \underset{\sim}{a}\cdot\underset{\sim}{b}$

$= k|\underset{\sim}{b}|^2 - |\underset{\sim}{a}|^2 + (k-1).\underset{\sim}{a}\cdot\underset{\sim}{b}$

(d) Simplify your result from (c) if $k = 1$, $\mathbf{a} = \mathbf{i} + 4\mathbf{j}$ and $\mathbf{b} = 3\mathbf{i} - 2\sqrt{2}\mathbf{j}$. (2 marks)

$\overrightarrow{OQ} \cdot \overrightarrow{PR} = 17 - 17 + 0$

$= \underline{0}$

$|\underset{\sim}{b}|^2$

$= 17$

$|\underset{\sim}{a}|^2$

$= 17$

(e) Explain the geometric significance of your result from (d). (2 marks)

diagonals $\perp$ $\therefore$ Rhombus

Question 18 **(7 marks)**

(a) The work done, in joules, by a force $\mathbf{F}$ Newtons in changing the displacement of an object $\mathbf{s}$ metres is given by the scalar product of $\mathbf{F}$ and $\mathbf{s}$. Calculate the work done when a force of 750 N moves an object a distance of 85 cm at an angle of 5° to the force.

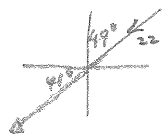
(2 marks)

$$W = \underset{\sim}{F} \cdot \underset{\sim}{s} = |\underset{\sim}{F}|\,|\underset{\sim}{s}| \cos\theta$$
$$= (750)(0.85)\cos 5^\circ$$
$$= \underline{635.1 \text{ J}}$$

(b) John is riding his jet ski travelling with velocity $\begin{pmatrix} 50 \\ -40 \end{pmatrix}$ km h^{-1}. To John, the wind appears to be coming from a bearing of 049° at 22 km h^{-1}. Determine the true speed of the wind.

(5 marks)

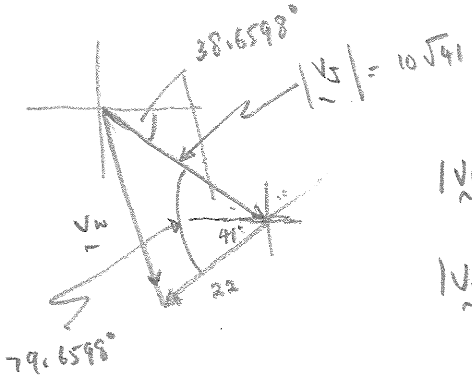
$$\underset{\sim}{V_J} = \begin{pmatrix} 50 \\ -40 \end{pmatrix}, \quad \underset{\sim}{V}_{W\,J} = \begin{pmatrix} -22\cos 41^\circ \\ -22\sin 41^\circ \end{pmatrix} = \begin{pmatrix} -16.604 \\ -14.433 \end{pmatrix}$$



$$\therefore \underset{\sim}{V_W} = \begin{pmatrix} 50 \\ -40 \end{pmatrix} + \begin{pmatrix} -16.604 \\ -14.433 \end{pmatrix} = \begin{pmatrix} 33.396 \\ -54.433 \end{pmatrix}$$

$$\therefore |\underset{\sim}{V_W}| = \underline{63.86 \text{ km h}^{-1}}$$

OR



$$|\underset{\sim}{V_W}|^2 = 4100 + 22^2 - 2.22.10\sqrt{41}\cos(79.6598)$$

$$|\underset{\sim}{V_W}| = \underline{63.86 \text{ km h}^{-1}}$$

Question 19 **(8 marks)**

(a) A high school has 5 male and 9 female volunteers from which to choose a debating team of 5 students. Determine the number of different teams that can be formed if

(i) there are no special requirements. (1 mark)

$\binom{14}{5} = \underline{2002}$

(ii) there must be a captain and a vice-captain. (2 marks)

$\overset{C}{\binom{14}{1}} \times \overset{VC}{\binom{13}{1}} \times \overset{\text{rest}}{\binom{12}{3}} = 40040$

(iii) there must be more females than males, but at least one male. (2 marks)

$\overset{M}{\binom{5}{1}}\binom{9}{4} + \overset{M}{\binom{5}{2}}\binom{9}{3} = 1470$

(b) Determine how many **different** numbers must be selected from the first 25 positive integers to be certain that at least one of them will be twice the other. (3 marks)

$\underline{1,2}$ $\underline{3,6}$ $\underline{4,8}$ $\underline{5,10}$ $\underline{7,14}$ $\underline{9,18}$ $\underline{11,22}$ $\underline{12,24}$

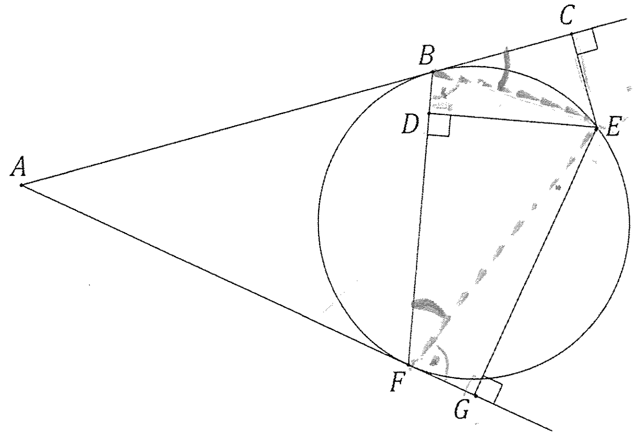
13 15 16 17 19 20 21 23 25

$\Bigg\}$ 17 $\underline{\text{PHoles}}$

$\therefore$ 18 numbers req'd by PH principle

Question 20 **(7 marks)**

In the diagram below, the tangents from point A touch the circle at B and F. Point E lies on the major arc BF and D lies on BF so that $DE \perp BF$. Points C and G lie on AB and AF extended respectively such that $EC \perp AC$ and $EG \perp AG$.



(a) Show that ΔBCE and ΔFDE are similar. (3 marks)

$\angle FDE = \angle BCE$ (90° given)

$\angle CBE = \angle DFE$ (alt seg thm)

$\therefore \Delta BCE \sim \Delta FDE$ (AA) —— 1

(b) Show that $DE^2 = CE \times GE$. (4 marks)

$\Delta FGE \sim \Delta BDE$ (AA) $\angle BDE = \angle EGF = 90°$ (given)
$\angle DBE = \angle EFG$ (alt seg thm)

$\therefore \frac{FE}{BE} = \frac{GE}{DE}$ | from 1 $\frac{BE}{FE} = \frac{CE}{DE}$

$\therefore \frac{DE}{GE} = \frac{CE}{DE}$

$DE^2 = GE \times CE$

End of questions